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Analyzing the efficiency of agricultural crop production by using mathematical models

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Abstract

Pepper (*Capsicum annuum L.*) is one of the main vegetables cultivated in our country. It covers about 3,000 hectares and occupies an important place in the structure of cultivation. Its cultivation in large areas is done because it firstly is used widely in our traditional cuisine and secondly, provides high incomes per area. The yield per area depends from the agricultural technology implemented, the type of culture and the cultivated areas. In our climate conditions, the pepper is much favorite and can be successfully cultivated in the open field and in protected environments. In the open field the pepper can be cultivated in three directions: For early production, semi-early (middle) and later production, while in protected environments it is cultivated during winter and early spring, where frosts and low temperatures do not allow its growth and production. It is known that for the production of each culture are used various inputs. In this study, in order to analyze the pepper production efficiency cultivated in the greenhouses of Lushnja district, DEA model was used (Data Envelopment Analysis). The DEA models represent one of the most important applications of mathematical programming in the agricultural economy. The chosen Output is the yield of pepper cultivated in greenhouses (q/are), while the inputs include: manure (q/are), fertilizer (q/dyn), liquid crystal manure (q/are), pesticides (q/are) and irrigation (m^3/are). From our analysis we found out that several units 4 gave a DEA efficiency of 100%, while other units (municipalities of Lushnja district) are less efficient. The aim of DEA model is not only to analyze but also to improve the efficiency of the inefficient units. Through this model, the weights of the composite unit which resulted more efficient than the non-efficient units were found, which means that this unit produces output greater than or equal to the output of the unit under study, requiring smaller or equal amounts of input.

Key Words: Efficiency, Agricultural production, DEA model, Composite unit, Pepper, Lushnja district.

1. Introduction

Vegetables are grown throughout the country in Albania by increasing household incomes average with more than 20%. Main crops grown in Albania are: tomato, cucumber, watermelon, pepper, legumes (fresh and dried), eggplants, carrots, onions, garlic, spinach and salads. The pepper crop takes about 15% of the area planted with vegetables, ranking second from tomato culture. Cultivation is large because, there is fairly wide use in our traditional cuisine and provides a good income for farmers as well.

Production of pepper, as well as in other vegetables, is strongly related to the right implementation of cultivation technology. Pepper has a relatively large demand for high temperature, compared with other plants. For a normal growth, pepper requires 18 - 25^{0}C. Humidity is also very important for a regular growth of this plant. Pepper has relatively a high growth in land with a neutral or slightly alkaline reaction. In case of cold and acid lands it gives low yields. Given the nutrient needs, the composting of pepper requires land with organic and mineral fertilizers. In the absence of nitrogen, in the process of growth vegetative organs of the plant will be destabilized and if procedures are not taken in time such as: additional fertilizer, leaf fall starts and the plant is weaken as a result, productivity will decrease. Even excessive nitrogen fertilization, when is unilateral, does not give good results after well develop vegetative measures, but slows the formation of fruit. Fertilization with phosphorus fertilizers helps strengthen the root system and, in particular, develops reproductive organs (buds, flowers, and fruits) also simultaneously helps in faster ripening of fruits. Phosphorus fertilization is done in the early stages of plant maturity, using the most appropriate fertilizers, such as granular after compared with those in powder form, easily dissolved and assimilated back into salts for plants. When a lack of phosphorus is the peppers is poorly developed, with few short branches and buds falling mass, and, consequently, reduced fruit. In the missing presence of potassium, pepper grows slowly and, at the end of the fruit leaves will appear brown spots, sometimes even caused the latest twist of their leaves and withering. Fertilization with potassium helps in collecting carbohydrates (sugar, starch) and increases plant resistance to low temperatures and disease. Throwing fertilizer calcium practiced only when the soil is acidic reaction (pH less than 6).

More recently are being wide spread crystalline fertilizer, who are more digestible and accessible to plants. Using them, a summary is given in Table 1. Guidance for fertilization rate of pepper in greenhouses.

Tabel 1. The yield of pepper and his culture using greenhouses factors in District of Lushnja

Production (kv/dy)	***Phenophases***	***Formula***	***Days***	***Kg/dy in a day***
80-90	Planting in the greenhouse until the blown	11-44-11	20	1.2
"	Blown until the connection of the fruit	19-19-19 + 1MgO +ME	20	1.8
"	Creation of the fruit until the early harvest	12-5-40 + 2MgO + ME	30	2.5
"	First harvest until the end of harvest	13-0-46	55	3-3.5

Depending on health conditions and stages of plant development, made interference with chemical manures, hoe, irrigation and chemical treatments to protect them from pests.

Irrigation: It is among the most important operations and more subtle, performed in the pepper plant. Pepper requires frequent watering, but a shortage of water, to maintain optimum moisture in the soil. Currently have been used some form of irrigation such as: *by flooding, with furrows, in the form of rain and drops.*

Pepper harvest time is determined by the purpose for which the product will be used, i.e. if pepper will be used for fresh consumption, processed will be used, or for export. Picking fresh consumption occurs when fruits have reached technical ripening. At this stage, fruit color, brightness and size characteristic for each cultivar is extremely important. In practice, several peppers clusters have, as well as fruits ripen at different times. When pepper is used for curing, baking harvested in early botanical. Also, the output will be used for drying and grinding (red pepper powder), botanical harvested after baking, when the fruits have fully taken the color red. Agriculturally implemented, yield per area depends by cultivar and cultivated areas. Usually taken in open field from 200 to 500 quintals / ha, while in protected environments from new hybrids taken over 800 q / ha.

Production is a process of transforming inputs into outputs. DEA (data envelopment analysis) has become one of the most widely used tools for efficiency analysis and as one of the most important recent applications of mathematical programming in economics [1]. DEA analyze the efficiency of company units (the agricultural activities in a farm) or the efficiency of the companies inside an industry (the farms of a region).The main idea consist on reducing the multi inputs and outputs situation for each units into a single <<virtual>> input and output. The goal in DEA is that an inefficient unit must be able to operate as efficiently as hypothetical composite unit, formed by a linear combination of efficient units [6]. This model has a wide use in various fields of production in economics.

2. Material and Methods

In this paper, DEA is applied to measure and improve the efficiency of each unit (municipalities of Lushnja district) for the pepper (*Capsicum annuum L.)*, production cultivated in greenhouses. The information for enabling this study was provided by:

- Interviews with experts and specialists in the field of agriculture Lushnja district.
- Studies conducted on this issue.
- Data collected in Regional Directory Agriculture and Food, Fier.

The selected Output is: production of pepper cultivated in greenhouses (q /are). The included Inputs are: Manure (q/are), Fertilizer (q/are), Liquid crystalline fertilizer (q/are) Pesticides (q/are), Irrigation (m^3/are).

DEA model was applied in order, minimizing the amount of inputs used to produce the same level of output (production culture of pepper). Once the problem arises as; a linear programming problem, with restrictions, imposed by the shape of the CCR model, solved with Solver Solution computer program. For inefficient units, Sealed linear combination of efficient units, which results in a composite unit, more efficient than inefficient unit. The data collected from surveys and questionnaires in 16 units (municipalities of Lushnja district), are arranged in Table 2.

Table 2:Information collected from 16 municipalities of Lushnja district

		OUTPUT	*INPUT*				
Unit	Municipalities	Yield	Manure	Fertilizer	Crystalline fertilizer	Pesticides	Irrigation
1	B.Lushnjë	108	85	1.8	1.4	0.06	55
2	B.Divjakë	102	85	1.9	1.3	0.65	50
3	Karbunarë	107	90	1.8	1.5	0.07	55
4	Fiershegan	107	80	1.9	1.5	0.07	50
5	Allkaj	102	100	1.7	1.6	0.07	40
6	Kruje	102	105	1.6	1.5	0.65	40
7	Bubullimë	96	90	1.4	1.5	0.07	45
8	Kolonjë	96	80	1.5	1.4	0.07	50
9	Gradisht	102	105	1.8	1.3	0.06	55
10	Rremas	83	95	1.7	1.5	0.06	45
11	Tërbuf	96	95	1.9	1.6	0.65	50
12	Dushk	98	100	1.6	1.5	0.07	45
13	Golem	93	105	1.8	1.7	0.65	55
14	Grabian	100	90	1.9	1.4	0.07	50
15	Hyzgjokaj	98	100	1.7	1.5	0.076	45
16	Ballagat	90	95	1.6	1.4	0.07	45

Source: Elaborated by the author

Determination of the decision variables- The efficiency unit assessed with DEA model is defined as follows [5, 7, and 8].

$$h(u,v) = \frac{\text{sum of the weighted outputs of the unit}}{\text{sum of weighted input unit}} = \frac{\sum_{r=1}^{s} y_r \times u_r}{\sum_{i=1}^{m} x_i \times v_i}$$

The problem in DEA model consists in assigning the weights v_i and u_r that represent the decision variables [6].

Defining the objective of our study- For each unit that will be assessed, the objective is the same:

$$Max\colon \sum_{r=1}^{s} y_r \times u_r$$

A unit is effective if it is between the limits of effectiveness. In this case, the study unit is assumed to be "the best practice." If h is less than 1, the unit is relatively inefficient. If any problem of LP (Linnear Programming) is resolved, the unit being considered or investigated choose the best possible weights for itself [3].

Setting the limits- It is impossible for any unit to have efficiency greater than 100%. So, if any problem of PL is resolved, the study units can not choose the weights that will give efficiency greater than 100% (including ourselves). For each unit is required that the amount of output weights of this unit is " ≤" to the sum of the weights of its inputs.

The first constraint: It corresponds to the first unit:

$$\sum_{r=1}^{s} y_r\, u_r \;\le\; \sum_{i=1}^{m} x_i\, v_i \qquad \textbf{or} \qquad \sum_{r=1}^{s} y_r\, u_r - \sum_{i=1}^{m} x_i\, v_i \le 0$$

These constraints are set for all units of the study.

The second constraint: To obtain an unrestricted solution, the sum of inputs weights for the review unit is equal to 1. $\sum_{i=1}^{m} x_i\, v_i = 1$.

***The third constraint*:** The non-negative condition $u_r \; and \; v_i \ge 0$, for all r and i. Units that are efficient will have a DEA efficiency score of 100% [7].

Model Application- To evaluate the efficiency of the unit 2, which is represented by Divjaka Municipality, we must solve the following LP problem. **Max:** $102u_1$ output weight for unit 2 [5].:

Constraints

$$\begin{cases} 102u_1 - 85v_1 - 1.9v_2 - 1.3v_3 - 0.65v_4 - 50v_5 \leq 0 \; efficiency\ constraint\ unit\ 2 \\ 108u_1 - 85v_1 - 1.8v_2 - 1.4v_3 - 0.06v_4 - 55v_5 \leq 0 \; efficiency\ constraint\ unit\ 1 \\ 107u_1 - 90v_1 - 1.8v_2 - 1.5v_3 - 0.07v_4 - 55v_5 \leq 0 \; efficiency\ constraint\ unit\ 3 \\ \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \\ \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \\ \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \\ 90u_1 - 95v_1 - 1.6v_2 - 1.4v_3 - 0.07v_4 - 45v_5 \quad \leq 0 \; efficiency\ constraint\ unit\ 16 \\ 85v_1 + 1.9v_2 + 1.3v_3 + 0.65v_4 + 50v_5 = 1 \qquad inputs\ conditions\ for\ unit\ 2 \\ \quad u_1, v_1, v_2, v_3, v_4, v_5 \geq 0 \qquad non-negativity\ constraint \end{cases}$$

This problem was elaborated by us with Excel (Solver Solution).

3. Findings and results

In the tables which stands in annexes we have demonstrated the results of the analysis presented by DEA efficiency, for unit 2 (Municipality Divjaka) and all other units taken into consideration. By applying parameters of Solver, we find the optimal solution for unit 2, which results in DEA efficiency equal to 93.3% (Tab 3). To complete the analysis of efficiency for other units should be changed content of the cell "Unit" (C24) manually in 1,3,4 .... 16 in order to optimize the Solver of the worksheet for each unit and their efficiencies in the column "DEA efficiency"[4].

Table 3. DEA analysis results for the units in the study

	A	B	C	D	E	F	G	H	I	J	K	L
1	Municipalities	Unit	output1	Input1	Input2	Input3	Input4	Input5	Weight	Weight	Diference	Efficiency
2	M	Number	Yield	Manure	Fertilizer	Crystalline Fertilizer	Pesticides	Irrigation	Output	Inpute	≤0	DEA
3	B. Lushnje	1	109	85	1.8	1.4	0.06	55	0,959	1,000	-0,041	**95.9%**
4	B. Divjake	2	102	85	1.9	1.3	0.65	50	0,897	1,000	-0,103	**93,3%**
5	Karbunare	3	107	90	1.8	1.5	0.07	55	0,941	1,059	-0,118	**90,2%**
6	Fiershegan	4	107	80	1.9	1.5	0.07	50	0,941	0,941	0,000	**100%**
7	Allkaj	5	102	100	1.7	1.6	0.07	40	0,897	1,176	-0,279	**100%**
8	Krutje	6	102	105	1.6	1.5	0.65	40	0,897	1,235	-0,338	**100%**
9	Bubullime	7	96	90	1.4	1.5	0.07	45	0,844	1,059	-0,214	**92%**
10	Kolonje	8	96	80	1.5	1.4	0.07	50	0,844	0,941	-0,097	**89,7%**
11	Gradishte	9	102	105	1.8	1.3	0.06	55	0,897	1,235	-0,338	**81,3%**
12	Remas	10	83	95	1.7	1.5	0.06	45	0,730	1,118	-0,388	**77,7%**
13	Terbuf	11	96	95	1.9	1.6	0.65	50	0,844	1,118	-0,273	**84,3%**
14	Dushk	12	98	100	1.6	1.5	0.07	45	0,862	1,176	-0,314	**89,9%**
15	Golem	13	93	105	1.8	1.7	0.65	55	0,818	1,235	-0,417	**74,1%**
16	Grabian	14	100	90	1.9	1.4	0.07	50	0,880	1,059	-0,179	**89,6%**
17	Hyzgjokaj	15	98	100	1.7	1.5	0.076	45	0,862	1,176	-0,314	**89,9%**
18	Ballagat	16	90	95	1.6	1.4	0.07	45	0,792	1,118	-0,326	**84,3%**
19												
20		Weights	0,0088	0,0118	0,0000	0,0000	0,0000	0,0000				
21												
22						Set Cell			Variable Cells			
23							Max: C25					
24		Unit	1				Variables: C20:H20					
25		Output	0,959				Constrains: C26=1					
26		Input	1,00		Constraint Cells		C20:H20>=0 K3:K18<=0					
27												
28												

	A	B	C	D	E	F	G	H	I
28		The formulas used for solving the Dea problem							
29	Cell	Formula				The formula's rang			
30	I3	SUMPRODUCT(C3:C3,$C$20:$C$20)					I3:I18		
31	J3	SUMPRODUCT(D3:H3,$D$20:$H$20)					J3:J18		
32	K3	I3-J3					K3:K18		
33	C25	INDEX(I3:I18,C24,1)							
34	C26	INDEX(J3:J18,C24,1)							

Source: Elaborated by the authors

But for a large number of units, this use of Efficiency Analysis would not be very adequate. Therefore we use the macro button in Excel, whose execution process is carried automatically, with the click of a button [2].As it is shown in Table 4, the results of this analysis for the 16 units taken into our study has proved that unit 2 (Divjake municipality) has a score of 93.3% efficiency, so it's proved inefficiency by DEA. To realize the goal of using DEA model in our study, we give an example of a composite unit, which has a higher efficiency than unit 2. In order to achieve this result, we must follow the following steps:

1. Solve the DEA problem for the study unit (unit2).
2. In the Solver Results dialog box, select the option Sensitivity Report.

In the results of the sensitivity, the absolute value of shadow prices (Shadow Prices) is the weighted composite unit, which results more efficient than the unit study. In Tab 5, is given the example of a composite unit, which is more efficient than unit 2 (Divjaka municipalities). In Tab 6, gives the average weights of about 83% for unit 4 (Municipality Fier-shegan), add 13% for Unit 5 (Allkaj) comprising a hypothetical unit (assumed). This composite unit produces an output equal to the output of unit 2 (Divjake municipality), 102 (q/dy) pepper by seeking small amounts of inputs (Tab 7). For any inefficient unit, we need to determine the linear combination of efficient units resulting in a more efficient unit consisting of inefficient unit.

Table 4: DEA efficiency analysis for the municipality of Divjaka

I4 =SUMPRODUCT(C4:C4;$C$20:$C$20)

	A	B	C	D	E	F	G	H	I	J	K	L
1	Municipalities	Unit	output1	Input1	Input2	Input3	Input4	Input5	Weight	Weight	Diference	Efficiency
2	M	Number	Yield	Manure	Fertilizer	Crystalline Fertilizer	Pesticides	Irrigation	Output	Inpute	≤0	DEA
3	B. Lushnje	1	109	85	1.8	1.4	0.06	55	0,997	1,065	-0,067	95.9%
4	B. Divjake	2	102	85	1.9	1.3	0.65	50	0,933	1,000	-0,067	93,3%
5	Karbunare	3	107	90	1.8	1.5	0.07	55	0,979	1,085	-0,106	90,2%
6	Fiershegan	4	107	80	1.9	1.5	0.07	50	0,979	0,979	0,000	100%
7	Allkaj	5	102	100	1.7	1.6	0.07	40	0,933	0,933	0,000	100%
8	Krutje	6	102	105	1.6	1.5	0.65	40	0,933	0,954	-0,021	100%
9	Bubullime	7	96	90	1.4	1.5	0.07	45	0,878	0,956	-0,078	92%
10	Kolonje	8	96	80	1.5	1.4	0.07	50	0,878	0,979	-0,101	89,7%
11	Gradishte	9	102	105	1.8	1.3	0.06	55	0,933	1,148	-0,215	81,3%
12	Remas	10	83	95	1.7	1.5	0.06	45	0,760	0,977	-0,218	77,7%
13	Terbuf	11	96	95	1.9	1.6	0.65	50	0,878	1,042	-0,163	84,3%
14	Dushk	12	98	100	1.6	1.5	0.07	45	0,897	0,998	-0,101	89,9%
15	Golem	13	93	105	1.8	1.7	0.65	55	0,851	1,148	-0,297	74,1%
16	Grabian	14	100	90	1.9	1.4	0.07	50	0,915	1,021	-0,106	89,6%
17	Hyzgjokaj	15	98	100	1.7	1.5	0.076	45	0,897	0,998	-0,101	89,9%
18	Ballagat	16	90	95	1.6	1.4	0.07	45	0,824	0,977	-0,154	84,3%
19												
20		Weights	0,0092	0,0042	0,000	0,000	0,000	0,013				
21												
22						Set Cell				Variable Cells		
23												
24		Unit	2									
25		Output	0,933									
26		Input	1,00			Constraint Cells						
27												

Max: C25
Variables: C20:H20
Constrains: C26=1
C20:H20>=0
K3:K18<=0

Source: Elaborated by the authors

Table 5. Sensitivity analysis for the municipality of Divjaka

Microsoft Excel 12.0 Sensitivity Report
Worksheet: Municipality of Divjaka

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$20	Weights Yield	0.009	0.000	102	1E+30	102
$D$20	Weights Manure	0.004	0.000	0	4.76635514	32
$E$20	Weights 1.6	0.000	0.000	0	0	1E+30
$F$20	Weights 1.4	0.000	0.000	0	0	1E+30
$G$20	Weights 0.07	0.000	0.000	0	0	1E+30
$H$20	Weights Irrigation	0.013	0.000	0	18.82352941	2.803738318

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$C$26	Input Yield	1.00	0.933	1	1E+30	1
$K$3	0.06 ≤0	-0.067	0.000	0	1E+30	0.06710727
$K$4	0.65 ≤0	-0.067	0.000	0	1E+30	0.066598882
$K$5	0.07 ≤0	-0.106	0.000	0	1E+30	0.106253177
$K$6	0.07 ≤0	0.000	0.830	0	0.060619977	0.160784314
$K$7	0.07 ≤0	0.000	0.130	0	0.018348624	0.279274327
$K$8	0.65 ≤0	-0.021	0.000	0	1E+30	0.020843925
$K$9	0.07 ≤0	-0.078	0.000	0	1E+30	0.077783427
$K$10	0.07 ≤0	-0.101	0.000	0	1E+30	0.100660905
$K$11	0.06 ≤0	-0.215	0.000	0	1E+30	0.214539908
$K$12	0.06 ≤0	-0.218	0.000	0	1E+30	0.217590239
$K$13	0.65 ≤0	-0.163	0.000	0	1E+30	0.163192679
$K$14	0.07 ≤0	-0.101	0.000	0	1E+30	0.101169293
$K$15	0.65 ≤0	-0.297	0.000	0	1E+30	0.296898831
$K$16	0.07 ≤0	-0.106	0.000	0	1E+30	0.105744789
$K$17	0.076 ≤0	-0.101	0.000	0	1E+30	0.101169293
$K$18	0.07 ≤0	-0.154	0.000	0	1E+30	0.153533299

Sensitivity Report | Dea efficiency2 | Sensitivity Report2 | Bashkia Divjake

Source: Elaborated by the authors

Table 6. The values of composition for the Municipality of Divjake

Municipalities	*Nr.*	*Yield*	*Manure*	*Fertilizer*	*Crystalline Fertilizer*	*Pesticides*	*Irrigation*	*Weights*	*Weights %*
Lushnjë	1	109	85	1.8	1.4	0.06	55	**0,000**	**0%**
Divjakë	**2**	102	85	1.9	1.3	0.65	50	**0,000**	**0%**
Karbunarë	3	107	90	1.8	1.5	0.07	55	**0,000**	**0%**
Fiershegan	4	107	80	1.9	1.5	0.07	50	**0,830**	**83%**
Allkaj	5	102	100	1.7	1.6	0.07	40	**0,130**	**13%**
Krutje	6	102	105	1.6	1.5	0.65	40	**0,000**	**0%**
Bubullimë	7	96	90	1.4	1.5	0.07	45	**0,000**	**0%**
Kolonjë	8	96	80	1.5	1.4	0.07	50	**0,000**	**0%**
Gradishtë	9	102	105	1.8	1.3	0.06	55	**0,000**	**0%**
Remas	10	83	95	1.7	1.5	0.06	45	**0,000**	**0%**
Terbuf	11	96	95	1.9	1.6	0.65	50	**0,000**	**0%**
Dushk	12	98	100	1.6	1.5	0.07	45	**0,000**	**0%**
Golem	13	93	105	1.8	1.7	0.65	55	**0,000**	**0%**
Grabian	14	100	90	1.9	1.4	0.07	50	**0,000**	**0%**
Hyzgjokaj	15	98	100	1.7	1.5	0.076	45	**0,000**	**0%**
Ballagat	16	90	95	1.6	1.4	0.07	45	**0,000**	**0%**
Composition values		**102**	**79,34**	**1.9**	**1.3**	**0.65**	**46,67**		
Additional Inputs used		**102**	**5,66**	**0**	**0**	**0**	**3,33**		

Table 7:. Improvement of efficiency for the municipality of Divjaka

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I4 =SUMPRODUCT(C4:C4;$C$20:$C$20)

	A	B	C	D	E	F	G	H	I	J	K	L
1	Municipalities	Unit	output1	Input1	Input2	Input3	Input4	Input5	Weight	Weight	Diference	Efficiency
2	M	Number	Yield	Manure	Fertilizer	Crystalline Fertilizer	Pesticides	Irrigation	Output	Inpute	≤0	DEA
3	B. Lushnje	1	109	85	1.8	1.4	0.06	55	1,069	1,141	-0,072	95.9%
4	B. Divjake	2	102	79,34	1.9	1.3	0.65	46,67	1,000	1,000	0,000	100%
5	Karbunare	3	107	90	1.8	1.5	0.07	55	1,049	1,163	-0,114	90,2%
6	Fiershegan	4	107	80	1.9	1.5	0.07	50	1,049	1,049	0,000	100%
7	Allkaj	5	102	100	1.7	1.6	0.07	40	1,000	1,000	0,000	100%
8	Krutje	6	102	105	1.6	1.5	0.65	40	1,000	1,022	-0,022	100%
9	Bubullime	7	96	90	1.4	1.5	0.07	45	0,941	1,025	-0,083	92%
10	Kolonje	8	96	80	1.5	1.4	0.07	50	0,941	1,049	-0,108	89,7%
11	Gradishte	9	102	105	1.8	1.3	0.06	55	1,000	1,230	-0,230	81,3%
12	Remas	10	83	95	1.7	1.5	0.06	45	0,814	1,047	-0,233	77,7%
13	Terbuf	11	96	95	1.9	1.6	0.65	50	0,941	1,116	-0,175	84,3%
14	Dushk	12	98	100	1.6	1.5	0.07	45	0,961	1,069	-0,108	89,9%
15	Golem	13	93	105	1.8	1.7	0.65	55	0,912	1,230	-0,318	74,1%
16	Grabian	14	100	90	1.9	1.4	0.07	50	0,980	1,094	-0,113	89,6%
17	Hyzgjokaj	15	98	100	1.7	1.5	0.076	45	0,961	1,069	-0,108	89,9%
18	Ballagat	16	90	95	1.6	1.4	0.07	45	0,882	1,047	-0,164	84,3%
19												
20		Weights	0,0098	0,0045	0,000	0,000	0,000	0,014				
21									Variable Cells			
22						Set Cell						
23												
24		Unit	2									
25		Output	1,000									
26		Input	1,00		Constraint Cells							

Max: C25
Variables: C20:H20
Constrains: C26=1
C20:H20>=0
K3:K18<=0

Source: Elaborated by the authors

4.Conclusion and recommendations

In this paper have been described how to analyze and improve the efficiency of the units included in the study (Lushnja municipalities), associated with the production of culture species cultivated in greenhouses. To achieve the goal was formulated and solved a mathematical problem PL, implemented in a spreadsheet and solve the Solver. The solution presented in Tab 3 and 4 shows that the units are operating 6, 7, 8, DEA efficiency 100%, other units are inefficient by DEA. For inefficient units, there is a linear combination of efficient units that turns a composite unit which produces at least the same output using the same or less than the unit inputs inefficient. For example, unit 2 (Divjaka Municipality) has a 93.3% efficiency result is therefore inefficient by DEA. The analysis of efficiency, referring to the report of the sensitivity results, the absolute value of shadow prices (Shadow Prices) are the weights of the composite unit , which results in more efficient the unit in study 2.

Through the use of the model, it was possible to improve the efficiency of this unit. Composition values were found to be used to get the same performance. Specifically found that the model for obtaining the same yield 102 (q/dy), should the amount of inputs used: 79.34 (q/dy) manure, 1.9 (q/dy) fertilizer, 1.3 (q/dy) crystalline fertilizer, 0.65 (q/dy) pesticides, 46.67 (m^3/dy) water. With the amount of input use, unit 2 (Divjaka Municipality) results with 100% efficiency. For any inefficient unit, we need to determine the linear combination of efficient units which result in a composite unit more efficient than the inefficient unit.

To conclude, the solution presented in the above tables show that several units are operating with a 100% DEA efficiency, while other units are less efficient. For inefficient units, there is a linear combination of efficient units that turns a composite unit which produces at least the same output using the same or fewer inputs than inefficient unit. The goal in DEA, is that an inefficient unit must be able to operate as efficiently as hypothetical composite unit, formed by a linear combination of efficient units. For example, unit 13 (Golem municipality) has a 74.1% efficiency, therefore it is inefficient by DEA. The analysis of efficiency, referring to the outcome of the sensitivity analysis (for the municipality of Golem), the absolute value of shadow prices (Shadow Prices) are the weighted composite unit, that results as more efficient as the study unit 13.

For any inefficient unit, we need to determine the linear combination of efficient units which result in a composite unit more efficient than the inefficient unit.

This model is also highly recommended for other similar analyses in different fields of economy and production industry.

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