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Logistic Equation and its Application as Forecasting Model of Vegetables Production in Greenhouses in Albania

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Abstract

Correct forecasting is of a great importance for the business and economy of the country. To comprehend the market and the economic system, mathematical models are used to describe and predict the future of situation. Agriculture is the spinal column of Albania's economic activity and the last 20 years free market experience has given a demonstration of the high correlation between agricultural progress and the economic development. Producing greenhouse-grown vegetables can result a beneficial activity, but it is a hard and complicated investment. The greenhouse technology is one of great innovation in agriculture. Agricultures methods must be combined with technical knowledge, marketing must be planned before harvest, and every phase of process should be well-managed. In this paper it is studied and applied the logistic growth model for forecasting the production of vegetables in greenhouse. The results of this paper show that the logistic S-shaped curve is a mathematical model to characterize the progress of innovation in agriculture. Also, the logistic equation can be used to describe and predict the production of vegetables in greenhouses in Albania.

Keywords: logistic model, vegetables in greenhouse, difference equation, forecasting.

1. Introduction

Accurate forecasting is essential and of a high importance for business community and the economic world. To understand the business environment and situations, and its possible effects it is necessary to look for a method which will give forecasts or overview of what might happen. Forecasts are necessary for planning, decision-making, understanding and implementing potential alternatives. In relation to this, both quantitative and qualitative methods must be used in explaining a particular situation. In most instances, quantitative methods of mathematics are used. Several economists, such as Malthus, Domar, and Verhaulst, utilized mathematical tools to explain their most profound contributions to the field of economics usually as a model or a formal mathematical statement of a presumed relationship between two or more variables. Majority of the models developed express the relationships between the variables in terms of equations and systems of equations. Some models involve differential equations in the representation of the relationships among economic variables which concern changes over periods of time. One of the many great results of ordinary differential equations is the derivation of the logistic equation

$$P(t) = \frac{K P_0 e^{rt}}{K+P_0(e^{rt}-1)}.$$

The S-shaped curves have been utilised in population dynamics and economic analyses (Kusharavy & De Guio, 2011). There are many examples of applying growth model for predicting the future of the different areas in business and economics, for instance: the future of principal energy recurces, the development of agriculture technologies, the substitution of transportation systems, the diffusion of innovation, the evolution of the aircraft industry, etc.

Logistic equations finds its use in the field of biological and social sciences as suggested by Bohner & Warth (2008) and Kooi, Boer & Kooijiman (1998). The study of Bohner & Warth (2008) was concentrated on the finding of the optimal harvesting policy and optimum level of population. Kooi, Boer & Kooijiman (1998) assumed that food chain models in the tropic level grow logistically. Bifurcation diagrams revealed that dynamic behaviours differ in large regions of the control parameter space due to dilution rate and concentration of resources.

Ramos (2013) applied the logistic equation in business and economics, in particular in product diffusion and market acceptance, inflation rate and prices of goods, purchasing power of Peso,

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employment and unemployment, inflation rate and purchasing power of peso. The results of his study indicated that problems that follow a logistic curve were product diffusion or market acceptance, inflation rate of goods, purchasing power of Peso, and employment and unemployment.

Agriculture is the spinal column of Albania's economic activity and the last 20 years free market experience has given a demonstration of the high correlation between agricultural progress and the economic development. The current agricultural situation is a mix of remarkable achievements and lost possibilities. If Albania has to take part of European Community, the agriculture productivity should be competitive with other European countries, which just have an advanced agriculture. A contemporary and effective technology can increase constantly the productivity, profitability, sustainability of farming systems. The greenhouse technology is one of great innovation in agriculture. The greenhouse is a structure where the climate conditions are so modified that can cultivate any vegetable in any location and independently of time. Greenhouses are constructions with roofs and walls made of glass or plastic that provides suitable climate background for plants, moreover it. It is used to protect the plants from the hostile weather such as wind, extreme temperature, insects and diseases.

In this paper, the focus is on the use of S-shaped curve to describe and to predict the future of the diffusion of the technology in greenhouses for vegetables production. The data are taken from the database of Albanian Institute of Statistics. The forecasting production levels generate from logistic functions were quite close to actual data.

2. Material and Methods

2.1. Malthusian and Logistic Growth

The linear model of growth of population (Malthus' Growth Model): $x_{n+1} = r\,x_n$ shows that *x* changes from the period time n to the period time n + 1 with coefficient r. If the value of r is greater than 1, then *x* gets bigger with successive iteration; whereas if the value of r is smaller than 1, then *x* decreases.

The Malthusian growth law is exponential over time but this does not agree with reality of facts. Growth normally takes place under restrictions and limits. The Logistic equation was originally developed in order to describe a growth and saturation in an environment with competition for limited resources.

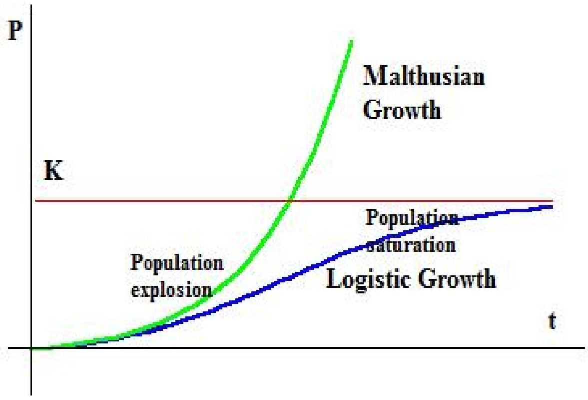

Figure 1. Malthusian and Logistic Growth

The S-shaped growth models can be described by an accelerated growth in the beginning (instinctive growth), an inflection point and after that the growth deceleration. The steady state is when the population approaches its Carrying Capacity.

Logistic model avoid unlimited growth that declines near a certain values. This becames true by the additional term (1 - x_n). The equation is modified in order that the maximum value to be 1. For example if the Carrying capacity is K,

$$x_{n+1} = r\,x_n (K - x_n)$$

Dividing by K, the new equation is:

$$\frac{x_{n+1}}{K} = r\frac{x_n}{K}\left(1 - \frac{x_n}{K}\right)$$

And taking $y_n = \frac{x_n}{K}$, and so $y_{n+1} = r\,y_n\left(1 - y_n\right)$ or $x_{n+1} = r\,x_n\left(1 - x_n\right)$. The factor (1 - x_n) brings growth slowdown, whenever x tends to 1 then (1- x_n) approaches to 0. The graph of x_{n+1} versus x_n is nonlinear (Fig. 2):

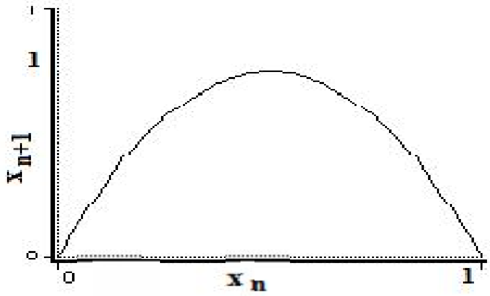

Figure 2. x_{n+1} versus x_n

The behavior of Logistic Equation depends on its control parameter r:

For 0 < r ≤1, 0 is an attractive fixed point, under iterations all points of [0 1] tend to 0.

For r > 1, the fixed point 0 is repelling and another fixed point 1 - $\frac{1}{r}$ lies in [0 1].

If $1 < r \leq 3$, the fixed point $1 - \frac{1}{r}$ is attractive and dynamic system is stable the basin of attraction is the entire open interval (0, 1).

A fundamentally different dynamics emerges for $3 < r < 4$, both fixed point become repelling and points of period two appear.

In the range $3 < r < 1 + \sqrt{6}$, all points in (0, 1) are attracted to the period two cycle, so we have period doubling bifurcations.

When r passes the second bifurcation point $1 + \sqrt{6}$ the period two points have become repelling and we get a four point attractor. Such successive bifurcations of each attractor into two,there are 4, 8, 16, 32, period cycle points. After an infinite period doubling there is a chaotic regime for values $3.57 < r \leq 4$

2.2. Applications of S-shaped Curve

At the beginning of 1960's, S-shaped curves were used for technological forecasting. Understanding the process of the diffusion of innovation is a critical issue in determining the extent to which innovative activities undertaken by market agents and government institutions, including the introduction of new products and processes, financing research and development activity, technology transfer, contribute to economic growth and social welfare, and allow various agents in the social and economic system to reduce economic and technological lags.

How the new technology does spreads in the agriculture business, in the production of vegetables in greenhouse? In general it is noticed the rule of the evolution: a slow rate of change, at the beginning; a fast increase, after; ending, the growth rate slows down. Three different stages can be illustrated by the figure 3:

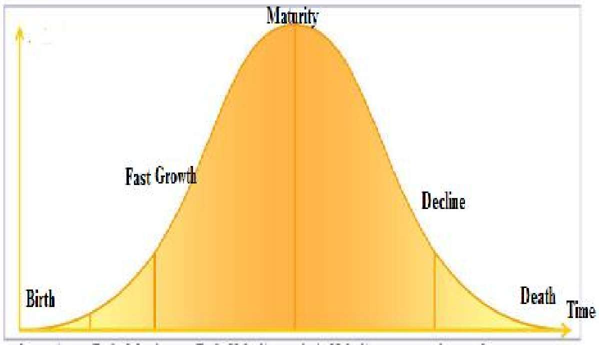

Figure 3. Bell-shaped curve

The principle of natural growth during a period of time can be described through periods of birth, growth, maturity, decline and death for every system. This structure is called *the life cycle of the system.*
In 1903, Gabriel Tarde suggested that the spread of innovation and the acceptance of new products tracks the S-shaped curve. More exactly, the penetration rate follows an S-shaped curve, and the penetration speed follows a Gaussian shaped curve.

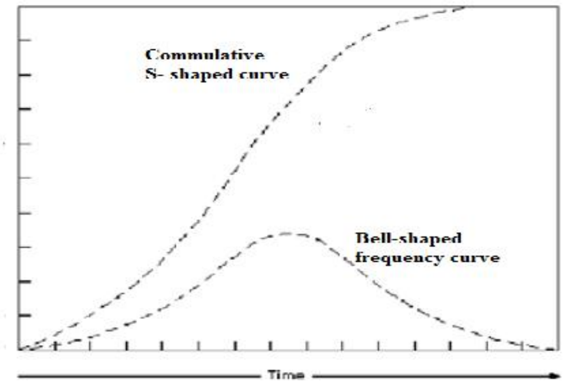

Figure 4. S-shaped curve and Bell-shaped curve

The bell shaped curves shows as model to stand for the rate of change and the cumulative growth (increasing in quantity) follows the S-shaped curves. The mathematical function that gives an S- curve is called *Logisti*c function. The Verhuls's differential equations is:

$$\frac{dP(t)}{dt} = r\,P(t)\left(1 - \frac{P(t)}{K}\right)$$

where is the rate of growth or the Malthusian parameter, and K is the maximum capacity.
The solution of this Bernulli Equation is:

$$P(t) = \frac{K\,P_0\,e^{rt}}{K + P_0(e^{rt} - 1)}$$

The parameters of the specific solution, which are determined using data points of equal time interval $\{(t_0, P_0), (t_1, P_1), (t_2, P_2)\}$ are given by the following formulae (Ramos, 2013):

$$r = \frac{1}{t_1}\ln\left|\frac{P_2(P_1 - P_0)}{P_0(P_2 - P_1)}\right|$$

and $K = \dfrac{P_1(P_0P_1 + P_1P_2 - 2P_0P_2)}{P_1^2 - P_0P_2}$ where $t_1 \neq 0$ and $P_0, P_1, P_2 \neq 0$.

2.3. The data

The data about the production of vegetables in greenhouses in Albania, are taken from the database of Albanian Institute of Statistics. The production levels in two type of greenhouses: heating greenhouses (with glasses and with plastic) and solar greenhouses (with glasses and with plastic) are shown in table 1. The Microsoft office Excel is used to obtain the parameters of the model and the predicted values from logistic function.

The greenhouse technology is one of great innovation in agriculture. The production of vegetables in heating greenhouses is increased from 1999 to 2013. A higher increase is in production of vegetables in solar greenhouses during the study period of time.

3. Results and Discussion

Table 1. Production of vegetables in Greenhouses (in tonne)

Year	Heating greenhouses			Solar greenhouses			Greehouses		
	with glasses	with plastic	Total	with glasses	with plastic	Total	Heating	Solar	***Total***
1999	1,051	310	1,361	6,742	23,886	30,628	1,361	30,628	**31,986**
2000	1,240	448	1,688	8,778	28,114	36,892	1,688	36,892	**38,580**
2001	714		714	5,090	28,622	33,712	1,167	33,712	**34,426**
2002	829	312	1,141	4,238	37,282	41,520	1,309	41,520	**42,661**
2003	1,155	455	1,610	5,404	40,125	45,529	1,610	45,529	**47,139**
2004	1,152	831	1,983	4,471	47,738	52,209	1,983	52,209	**54,192**
2005	527	2,053	2,580	5,516	50,566	56,082	2,580	56,082	**58,662**
2006	1,620	2,850	4,470	4,254	48,817	53,071	4,470	53,071	**57,541**
2007	1,279	3,975	5,254	4,042	47,018	51,060	5,254	51,060	**56,314**
2008	1,437	5,600	7,037	3,426	50,749	54,175	5,627	54,175	**61,212**
2009	1,173	3,834	5,006	3,958	51,357	55,315	5,006	55,315	**60,321**
2010	1,330	3,262	4,592	3,914	57,811	61,725	4,592	61,725	**66,317**
2011	1,336	4,768	6,104	3,813	61,053	64,856	6,104	64,856	**70,960**
2012	2,373	3,019	5,392	2,831	70,284	73,115	5,392	73,115	**78,507**
2013	1,400	3,450	4,850	5,900	74,500	80,400	4,850	80,400	**85,250**

Source: INSTAT database http://www.instat.gov.al/al/themes/agriculture,-forestry-and-fishery.aspx?tab=tabs-5

The figure 5 indicates a S-shaped curve for production of vegetables in heating greenhouses with plastic and a lower level of production in heating greenhouses with glasses compared to heating greenhouses with plastic since 2004.

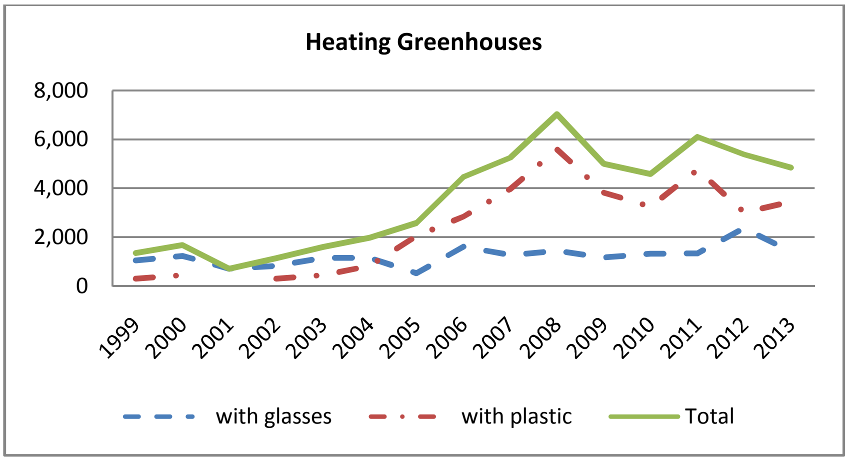

Figure 5. Production of vegetables in Heating Greenhouses

The figure 6 indicates a S-shaped curve for production of vegetables in solar greenhouses with plastic and an almost constant production level in solar greenhouses with glasses.

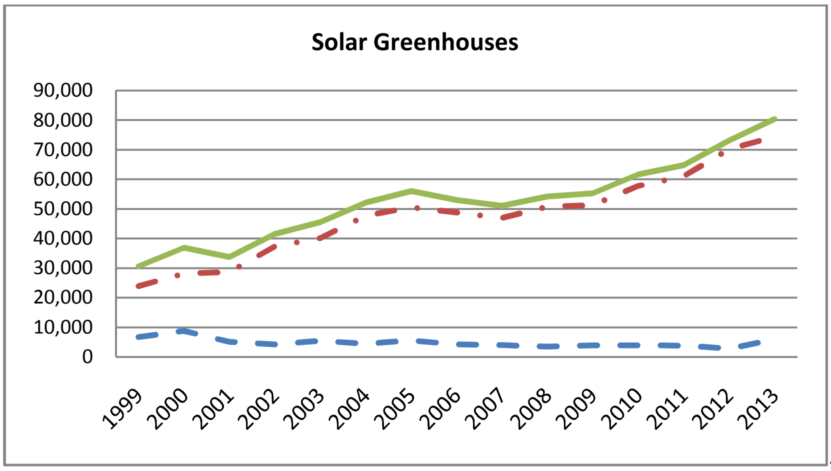

Figure 6. Production of vegetables in Solar Greenhouses

The figure 7 for total production in greenhouses indicates a S-shaped curve for production in solar greenhouses and a small proportion of production of vegetables in heating greenhouses.

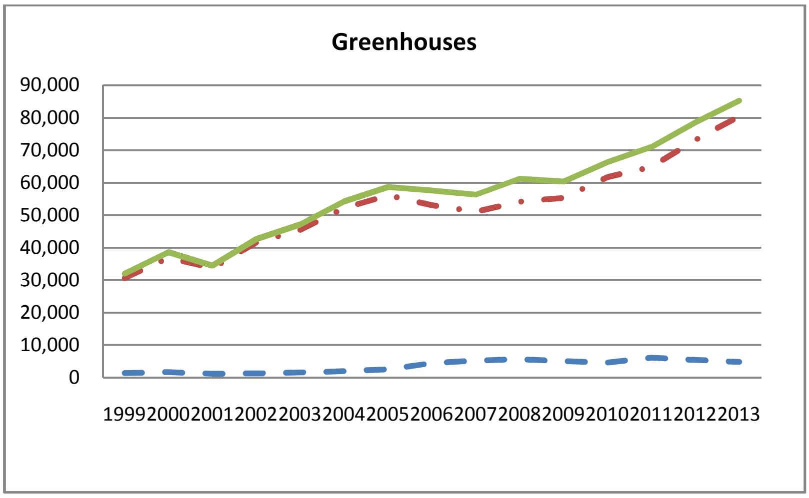

Figure 7. Production of vegetables in Greenhouses by type of greenhouse

It is clear that forecasts are necessary for planning, making right decisions and implementing marketing strategies. Applying logistic model for forcasting needs the exact quantification of the growth process that is to find the value of the ceiling and the rate of the growth (control parameter). To obtain the values of r and K, a time interval of 7 years was taken, and the three point of time were $t_0 = 0$ (year = 1999), $t_1 = 7$ (year = 2006) and $t_2 =$ 14 (year = 2013). Table 7 indicate the actual production in heating greenhouses with plastic, solar greenhouses with plastic, total greenhouses for period 1999-2013 and predicted production in greenhouses for the time interval of 7 years ($t_1 = 7$). For the time interval of 7 years, the value of two parameters r anf K were: r = 0.55036 and K = 3,466 for production in heating greenhouses with plastic, r = 0.158256 and K = 100,603 for production in solar greenhouses with plastic, and r = 0.12848 and K = 127,291 tonne for production of vegetables in total greenhouses.

Table 2. S-shaped forecasting of production of vegetables in greenhouses

	Heating greenhouses with plastic		Solar greenhouses with plastic		Total Greenhouses	
Year	Actual data	Logistic	Actual data	Logistic	Actual data	Logistic
1999	310	310	23,886	23,886	31,986	31,986
2000	448	504	28,114	26,887	38,580	35,160
2001		790	28,622	30,117	34,426	38,522
2002	312	1,173	37,282	33,559	42,661	42,058

2003	455	1,629	40,125	37,186	47,139	45,751
2004	831	2,100	47,738	40,966	54,192	49,581
2005	2,053	2,520	50,566	44,858	58,662	53,520
2006	2,850	2,849	48,817	48,817	57,541	57,541
2007	3,975	3,081	47,018	52,795	56,314	61,611
2008	5,600	3,233	50,749	56,742	61,212	65,698
2009	3,834	3,327	51,357	60,609	60,321	69,769
2010	3,262	3,385	57,811	64,354	66,317	73,789
2011	4,768	3,419	61,053	67,937	70,960	77,728
2012	3,019	3,438	70,284	71,327	78,507	81,557
2013	3,450	3,450	74,500	74,500	85,250	85,250
2014		3,457		77,441		88,785
2015		3,461		80,142		92,146
2016		3,463		82,601		95,319
2017		3,464		84,822		98,296
2018		3,465		86,816		101,072
2019		3,465		88,593		103,646
2020		3,466		90,168		106,020
2021		3,466		91,558		108,200
2022		3,466		92,779		110,193
2023		3,466		93,847		112,007
2024		3,466		94,779		113,652
2025		3,466		95,589		115,139
2026		3,466		96,291		116,480
2027		3,466		96,899		117,685
2028		3,466		97,424		118,766
2029		3,466		97,877		119,732
2030		3,466		98,267		120,596

The actual and predicted values for production in heating greenhouses with plastic shown in figure 8 indicate that the maximum capacity or maximum production level is almost achieved. This means that in the near future is not predicted to build this type of greenhouses and to increase the level of production beyond the maximum level of production of 3,466 tonne.

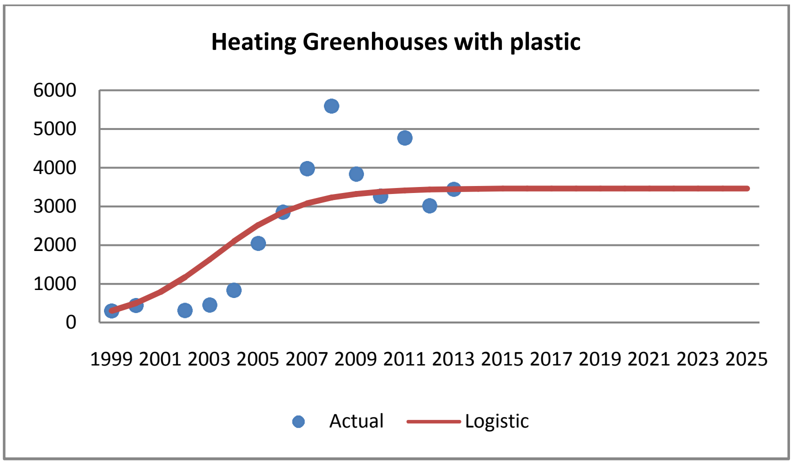

Figure 8. Predicted production in heating greenhouses with plastic from logistic function

The actual and predicted values for solar greenhouses with plastic shown in figure 9 indicate that the maximum capacity is not achieved. The maximum capacity of production of 100,603 tonne, if the same conditions holds, can be achieved after the 2030 year.

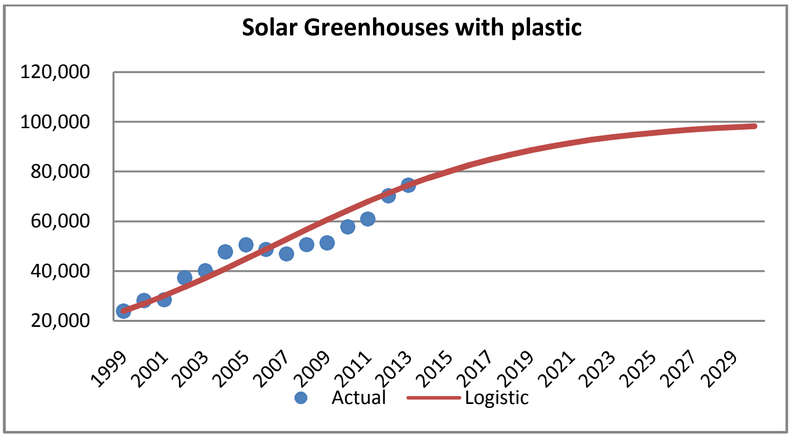

Figure 9. Predicted production in solar greenhouses with plastic from logistic function

Figure 9 shows how close the logistic curve with the curve of actual values of production in solar greenhouses with plastics. Notice that the logistic curve intersects the curve of actual level of production.

Also, the actual and predicted values for production in greenhouses shown in figure 10 indicate that the maximum capacity is not achieved yet. The actual level of production is half of the maximum production. The maximum level of production of 127,291 tonne, if the same conditions hold, can be achieved after year 2030.

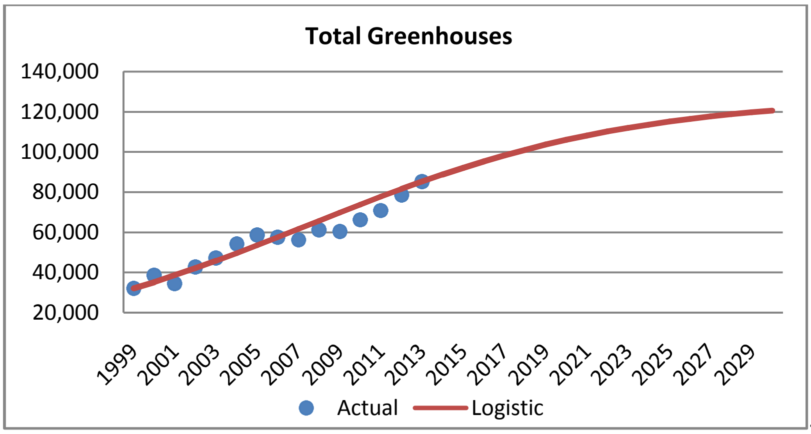

Figure 10. Predicted production in greenhouses from logistic function

Figure 10 shows that the logistic curve predict the values quite close to the original values of production of vegetables in greenhouses.

4. Conclusions

The logistic function can be used as a good forecasting model. It can significantly determine at certain degree of accuracy how effective are the measures implemented to a certain concern such as greenhouses for vegetable production. Based on the findings, the logistic functions give close approximations of the actual values. But, the appropriate choice of time interval should be considered as it affects greatly the predicted values from the logistic equation.

Greenhouses for vegetable production is destined to play more and more a principal part in the Albanian climate environment as a way for sustainable production of vegetables besides the best regulation of product quality and reliability, compatible with the market request, standards and regulations. In addition to supply the local market, the production of greenhouse vegetables is significant for its export potential and represents an important part in the balance of foreign trade. Glass and plastic greenhouses are a new ability of growing vegetables, but the statistics show that poly greenhouses give a better investment profits than the glass ones (5).

The object of the paper was to improve the results of forecasting of greenhouse technology adaption in Agriculture applying logistic S-shaped curve. It is concluded that application of the rule of natural growth with a logistic S-curve, can improve fundamentally the precision of long-term forecasting.

The forecasting results from logistic functions indicate that the maximum level of production of vegetables in heating greenhouses with plastic is 3466 tonne and can be achieved within a period of 5 years, the maximum level of production in solar greenhouses with plastic is 100,603 tonne and it can be achieved after year 2030, whereas the maximum level of production in total greenhouses is 127,291 tonne and also it can be achieved after year 2030.

Other important areas where the logistic function can be used are Information and communication technology problems such as internet diffussion, inflation rate of goods, employment, climate and disaster management, and sustainability problem.

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