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The impact of the influence of four food factors and their combination in milk production in cattle and average weight of newborn calves by the mathematical modeling and Bernstein polynomial approximation

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Abstract

In this paper published economic analysis of the impact of the four food factors in the milk production and average weight of newborn calves and use of the mathematical models in the livestock sector. The main purpose of this paper is the use of modern methods in economic analysis of use of resources in a farming complex. Livestock development in general and milk production in particular is closely linked to many factors which are the main breeding. This study analyzes the economic impact of four components ration (wet food, dry food, concentrate and mineral salts) used for milk production and average weight of newborn calves. The study was conducted in Lushnje district. Are analyzed and processed data feeding phases (1-up in 150 days lactation, 2-over 150 days lactation and 3- period of drying), milk production and average weight of newborn calves for a period of 6 years. This study used approaches through approximation with the method of undetermined coefficients to solve systems of constraints and production function to analyze the impact of the three components of the ration on milk production, weight of calves and quantity of manure. This study confirms that balanced nutrition is a major factor in increasing economic efficiency of farms. In proportion to the daily ration we have to be awarded: 56.8 % wet food, dry food 29.43% , 13.56 % koncetrat and 0.21 % mineral salts of 48.8 kg food per day.

Key words: optimal structure; milk production; nutritive factors; Bernstein polynomial approximation; the food ration.

1. Introduction

This study publishes the economic analyses of the impact of four nutrition factors – moisture food, dry food, concentrate and mineral salts — in 2 outputs: milk yield and the average weight Milk production mainly from the caws varies in different areas of Albania. Nowdays the sustainable development of agricultural farms and especially of livestock farms requires optimization of production and at the same time the continuous analyses of economic and techinical impact factors of inborn calves, as well as the use of production function in agricultural sector. The production function used during the study is that of Cobb-Douglas and it aims to analyse the impact of the four impact nutrition factors on caw's milk production and on the average weight of newborn calves. To be successful, dairy producers must master all aspects of dairy management. Proper dry cow nutrition and management is critical, since decisions made during this period will have a tremendous impact on milk production and health during the next lactation [9]. This study proves once more that using cattle volume nutrition system in our country's conditions makes up the primary factor for increasing the economic effectivity of farms. At the end of the study it is proved that the maximal income and profits in cattle farms are reached at the same point of route expansion where the cost is minimal.

2. Material and Methods

- The production function forms for milk and the average weight of inborn calves are determined.
- The suitability of selected models is proved.

- The optimal combination of inputs (food ration structure) to maximize gain and minimize costs is
- discovered.
- It is used the approximation method by using the Bernstein polynomial to solve the system of margins and
- to determine the optimal structure of milk production.
- It is used the method of linear regression to determine the parameters of the model through the packet of
- econometric computerized programmes SPSS.
- After deciding on options through estimates it is calculated the correct solution based on MAPLE program.

The livestock complex studied for this purpose was “Agrotex” in the field area of Tre Ura village, Grabian commune, Lushnje disctrict. The data of feeding through stages was analysed and processed (1- up to 150 days of lactation, 2- over 150 days of lactation and 3- dry period), as well as data on milk yield and the average weight of newborn calves. These data were analysed for a six-year period. In order to realize a more accurate dependance of the newborn calves’ weight and milk quantity from inputs (food) it is procedeed according months. The average values of milk yield and average quantity of food were grouped thoughout a year (per months) according to the 3 stages of caw treatment, as well as the average weight of newborn calves. After data processing there were built concreate functions, milk yield analyses and the average weight of newborn calves was undertaken after data processing in relation to the three production functions (moisture, dry, concentrate food and mineral salts). The production function was requested in the following form $y = Ax_1^{\alpha} x_2^{\beta} x_3^{\gamma} x_4^{\delta}$

The appeal of the Cobb-Douglas type of function rests largely with its simplicity [4].

Linear regression method was used to determine $\log A, \alpha, \beta \ \delta \ dhe \gamma$ through the econometric computerized programmes SPSS, from which resulted that the models are suitable. We can

save predicted values, residuals, and other statistics useful for diagnostics. Each selection adds one or more new variables to your active data file [11]. It came out that the models were appropriate. The presence of association does not necessarily imply causation. Statistical tests can only establish whether or not an association exists between variables [15]. It is confirmed the hypothesis for the importance general regression and shown that at least one of the variables provides information for prognosis of y [2]. Based on these data, the folllowing production functions were built:

$$y_1 = 34.05578788 \cdot x_1^{0.243} \cdot x_2^{0.143} \cdot x_3^{0.218} \cdot x_4^{0.025}$$
$$y_2 = 0.27117254 \ x_1^{0.273} \cdot x_2^{0.142} \cdot x_3^{0.17} \cdot x_4^{0.025}$$

where x_1, x_2, x_3, x_4, y_1 and y_2 show respectively the amount of moisture, dry food, concentrate mineral salts, average milk production and the average weight of newborn calves.

3. Results and Discussion

3.1. Profit maximization

As in the single product case, the mathematics of profit maximization can be approached as either a constrained optimization problem or unconstrained optimization problem formulated by substituting the explicit form of the production function into the profit function for x [1]. For profit maximization we have asserted the profit function:

$$F = p_1 f(x_1, x_2, x_3, x_4) + p_2 g(x_1, x_2, x_3, x_4) - r_1 x_1 - r_2 x_2 - r_3 x_3 - r_4 x_4$$

Also, we have asserted the necessary conditions for profit maximization.

$$\frac{\partial F}{\partial x_1} = p_1 f'_{x_1}(x_1, x_2, x_3, x_4) + p_2 g'_{x_1}(x_1, x_2, x_3, x_4) - r_1 = 0$$
$$\frac{\partial F}{\partial x_2} = p_1 f'_{x_2}(x_1, x_2, x_3, x_4) + p_2 g'_{x_2}(x_1, x_2, x_3, x_4) - r_2 = 0$$
$$\frac{\partial F}{\partial x_3} = p_1 f'_{x_3}(x_1, x_2, x_3, x_4) + p_2 g'_{x_3}(x_1, x_2, x_3, x_4) - r_3 = 0$$
$$\frac{\partial F}{\partial x_4} = p_1 f'_{x_4}(x_1, x_2, x_3, x_4) + p_2 g'_{x_4}(x_1, x_2, x_3, x_4) - r_4 = 0$$

Thus we have to solve the system (*)

$$\begin{cases} p_1\alpha\, y_1 + p_2 a y_2 = r_1 x_1 \\ p_1\beta\, y_1 + p_2 b y_2 = r_2 x_2 \\ p_1\gamma\, y_1 + p_2 c y_2 = r_3 x_3 \\ p_1\delta\, y_1 + p_2 d y_2 = r_4 x_4 \\ y_1 = A x_1^{\alpha} x_2^{\beta} x_3^{\gamma} x_4^{\delta} \\ y_2 = B x_1^{a} x_2^{b} x_3^{c} x_4^{d} \end{cases} \qquad (*)$$

Technically it is difficult to find the accurate solution of the system (*), thus we have found close solutions with a satisfying estimate through the method Bernstein polynomials, that has made possible determining the areas where y_1 and y_2 move. Theorem presents a method for using polynomials to approximate functions that cannot be evaluated [7]. To solve systems of equations numerically, one can use the f solve (float solve) command [10]. These areas have served as options to find the accurate solution of the system (*) based on MAPLE programme. Remember that:

$\alpha = 0.243, \beta = 0.143, \gamma = 0.218, \delta = 0.025, a = 0.273, b = 0.142, c = 0.17, d = 0.025,$

$p_1 = 47,\ p_2 = 350,\ r_1 = 7.8,\ r_2 = 8.8,\ r_3 = 28.8,\ r_4 = 215$.

Let be the continuous function in the segment [0,1]. Consider the Bernstein polynomial for the function f (x) which is $B_n(f)(x) = \sum_{k=o}^{n} f\left(\frac{k}{n}\right)\cdot C(n,k)x^k(1-x)^{n-k}$. Prove that the functional string $\{B_n(f)(x)\}$ converges uniformly to $f(x)$, thus. $\lim_{n\to\infty} B_n(f)(x) = f(x)$ or $f(x) \approx B_n(f)(x)$, $\forall x \in [0,1]$ where $\frac{k}{n}$ are the division points [0,1] per $k \in \{0,1,2,3,...,n\}$.

If the function then with the function side $f(x)$ is continuous in the segment $[a,b]$, then by the function $g: x \to (b-a)x + a$ we have that $[0,1] \to [a,b]$. The separation points in the segment $[a,b]$ will be

$$\xi_k = (b-a)\frac{k}{n} + a$$

per $k \in \{0,1,2,3,...,n\}$. Then $B_n(f)(x) = \sum_{k=o}^{n} f(\xi_k)\cdot C(n,k)\left(\frac{x-a}{b-a}\right)^k\left(\frac{b-x}{b-a}\right)^{n-k}$ or

$$B_n(f(x)) = \frac{1}{(b-a)^n}\sum_{k=0}^{n} f(\xi_k)\cdot C(n,k)(x-a)^k(b-x)^{n-k} .$$

Thus, the production function for milk is approximated with the polynomial function as follows

$$\begin{aligned} y_1 \approx\ & 0.2128486742\cdot 10^{-23}[9.626929531(11250-x_1)^4 + 38.528681(x_1-11150)(11250-x_1)^3 + \\ & 57.82441262(x_1-11150)^2(11250-x_1)^2 + 38.57050049(x_1-11150)^3(11250-x_1) + \\ & 9.647839344(x_1-11150)^4][3.385093486(5150-x_2)^4 + 13.54993918(x_2-5050)(5150-x_2)^3 + \\ & 20.33919619(x_2-5050)^2(5150-x_2)^2 + 13.56894914(x_2-5050)^3(5150-x_2) + \\ & 3.394598647(x_2-5050)^4][5.45614685(2500-x_3)^4 + 21.87394679(x_3-2400)(2500-x_3)^3 + \\ & 32.88436475(x_3-2400)^2(2500-x_3)^2 + 21.97148372(x_3-2400)^3(2500-x_3) + \\ & 5.504918858(x_3-2400)^4][1.094100765(38.5-x_4)^4 + 4.377891908(x_4-36.5)(38.5-x_4)^3 + \\ & 6.569041902(x_4-36.5)^2(38.5-x_4)^2 + 4.380811648(x_4-36.5)^3(38.5-x_4) + \\ & 1.095560885(x_4-36.5)^4] \end{aligned}$$

Similarly, it is used to find a polynomial function that approximates the function of meat production.

$$y_2 \approx 0.1694828344\,10^{-25}[12.73227584(11250-x_1)^4 + 50.96025208(x_1-11150)(11250-x_1)^3 + 76.4871524(x_1-11150)^2(11250-x_1)^2 + 51.02239784(x_1-11150)^3(11250-x_1) + 12.76334882(x_1-11150)^4][3.356351028(5150-x_2)^4 + 13.43482179(x_2-5050)(5150-x_2)^3 + 20.16629962(x_2-5050)^2(5150-x_2)^2 + 13.45353834(x_2-5050)^3(5150-x_2) + 3.365709484(x_2-5050)^4][3.755224547(2500-x_3)^4 + 15.04738343(x_3-2400)(2500-x_3)^3 + 22.61046452(x_3-2400)^2(2500-x_3)^2 + 15.09968112(x_3-2400)^3(2500-x_3) + 3.78137546(x_3-2400)^4][1.094100765(38.5-x_4)^4 + 4.377891908(x_4-36.5)(38.5-x_4)^3 + 6.569041902(x_4-36.5)^2(38.5-x_4)^2 + 4.380811648(x_4-36.5)^3(38.5-x_4) + 1.095560885(x_4-36.5)^4]$$

After determining the coefficients, the system was solved (*), by replacing the respective estimates in the last two equations, i.e. in solving the following system

$11.421\ y_1 + 95.55\ y_2 = 7.8\ x_1$

$6.721\ y_1 + 49.7\ y_2 = 8.8\ x_2$

$10.246\ y_1 + 59.5\ y_2 = 28.8\ x_3$

$1.175\ y_1 + 8.75\ y_2 = 215\ x_4$

Accurate solutions through MAPLE programme which are the following values

(x_1 = 10124.36297, x_2 = 5246.24891, x_3 = 2417.373417, x_4 = 37.5535413,

y_1 = 6523.527045, y_2 = 46.72766884) .

So, the area where inputs and outputs vary are as below :

$x_1 \in [10000,13000]$, $x_2 \in [4000,6000]$, $x_3 \in [2000,3000]$, $x_4 \in [25,45]$, $y_1 \in [5000,7000]$, $y_2 \in [30,60]$. Through MAPLE, the accurate results are:

x_1= 10117.25967, x_2= 5242.567396,x_3 = 2415.67649,x_4 = 37.52718868,

y_1= 6518.94196, y_2= 46.69585859 .

Let's see the fulfillment of necessary conditions for profit maximization. In order to do this we need to determine the sign of Hessian determineras well as its main minors. The necessary conditions for profit maximization require that main minors leading Hessian determiner shpuld be alternated in sign, starting with a negative sign to the first order minor. That is:

We have proved that $|H_1| < 0$, $|H_2| > 0$, $|H_3| < 0$ dhe $|H_4| > 0$ i.e. profit maximization will be achieved for the values x_1, x_2 and x_3 that solve the system (*).

3.2. Conditioned cost minimization

The following function are given

$y_1 = Ax_1^{\alpha} x_2^{\beta} x_3^{\gamma} x_4^{\delta}$ dhe $y_2 = Bx_1^{a} x_2^{b} x_3^{c} x_4^{d}$ ose

$y_1 = f(x_1, x_2, x_3, x_4)$ dhe $y_2 = g(x_1, x_2, x_3, x_4)$.

Lets sign r_1, r_2, r_3 and r_4 accordingly input prices x_1, x_2, x_3 and x_4, and then with p_1 and p_2 output prices respectively y_1 and y_2. If S is the set of all points (x_1, x_2) that satisfy a number of equations we have the Lagrange an problem of maximizing or minimizing a function subject to equality constraints [5]. Each solution (x_1, x_2, λ) gives a candidate (x_1, x_2) for an extreme point. We can finally compare the values of function at these candidate points to ascertain where its

maximum and minimum values on S are attained [8]. We can deal with optimization problems with two or more constraints simply by using two or more Lagrange multipliers [6].

For the minimum of cost we have formed Lagrange function
LC: $LC = r_1x_1 + r_2x_2 + r_3x_3 + r_4x_4 + \lambda_1[y_1 - f(x_1,x_2,x_3,x_4)] + \lambda_2[y_2 - g(x_1,x_2,x_3,x_4)]]$ where λ_j is Lagrange multiplier, accompanied with the function of production for y_j, j=1,2.

And we have expressed the necessary conditions for the minimum of LC :

$$\frac{\partial LC}{\partial x_1} = r_1 - \lambda_1 f'_{x_1}(x_1,x_2,x_3,x_4) - \lambda_2 g'_{x_1}(x_1,x_2,x_3,x_4) = 0$$
$$\frac{\partial LC}{\partial x_2} = r_2 - \lambda_1 f'_{x_2}(x_1,x_2,x_3,x_4) - \lambda_2 g'_{x_2}(x_1,x_2,x_3,x_4) = 0$$
$$\frac{\partial LC}{\partial x_3} = r_3 - \lambda_1 f'_{x_3}(x_1,x_2,x_3,x_4) - \lambda_2 g'_{x_3}(x_1,x_2,x_3,x_4) = 0$$
$$\frac{\partial LC}{\partial x_4} = r_4 - \lambda_1 f'_{x_4}(x_1,x_2,x_3,x_4) - \lambda_2 g'_{x_4}(x_1,x_2,x_3,x_4) = 0$$
$$\frac{\partial LC}{\partial \lambda_1} = y_1 - f(x_1,x_2,x_3,x_4) = 0$$
$$\frac{\partial LC}{\partial \lambda_2} = y_2 - g(x_1,x_2,x_3,x_4) = 0$$

This determinant often referred to as a bordered Hessian, where the bar on top symbolizes the border [12]. And after we have assessed the necessary conditions for the minimum through bordered Hessian determiner we have proved that the cost function has a minimum for the values of x_1 and x_2 shown in the equations (*).

$$\begin{cases} 0.243\cdot 6518.94196\lambda_1 + 0.273\cdot 46.69585859\lambda_2 = 7.8x_1 \\ 0.143\cdot 6518.94196\lambda_1 + 0.142\cdot 46.69585859\lambda_2 = 8.8x_2 \\ 0.218\cdot 6518.94196\lambda_1 + 0.17\cdot 46.69585859\lambda_2 = 28.8x_3 \\ 0.025\cdot 6518.94196\lambda_1 + 0.025\cdot 46.69585859\lambda_2 = 215x_4 \\ 34.05578788\cdot x_1^{0.243}\cdot x_2^{0.143}\cdot x_3^{0.218}\cdot x_4^{0.025} = 6518.94196 \\ 0.27117254\cdot x_1^{0.273}\cdot x_2^{0.142}\cdot x_3^{0.17}\cdot x_4^{0.025} = 46.69585859 \end{cases}$$

These estimates were used in the MAPLE programme. It is found that
x_1 = 10117.25758 , x_2 = 5242.567226 , x_3 = 2415.677108, x_4 = 37.52718711 (a)
λ_1 = 47.00006424 , λ_2 = 349.9907429 .

Lets assess the necessary conditions for the minimum through Hessianit determiner with the margin. $|\overline{H}| = 0.6816014792\cdot 10^{-9} > 0$. $|\overline{H_1}| = 0.1140371348\cdot 10^{-9}$ > 0. Then we proved that cost function has a minimum for x_1 , x_2 , x_3 and x_4 shown in equation (a). The values y_1 and y_2 are in the production level for which the cost will be minimal. The change of input values in 365 days for profit maximization and cost minimization are almost unnoticed. This shows that profit maximization and cost minimization is achieved for the same input values.

We have proved that profit maximization and cost minimization is x_1 =10117.25967 moisture food, x_2 = 5242.567396 dry food, x_3 = 2415.67649 kg concentrat and x_4 =37.52718868 mineral salts. In this case the amount of milk produced by a caw will be 6518.942 kg while the average weight of newborn caws will be 46.69586 kg , It is proved that the maximum profit and maximum revenue achieved in the same point of the expansion path where the cost is minimal [3].

Food amount per year :
Moisture food quantity = 10117.26 kg .
Dry food quantity = 5242.57 kg .
Concentrate food = 2415.68 kg .
The amount of mineral salts = 37.53 kg

Cost = 10117.26 · 7.8 + 5242.57.73 · 8 + 2415.68 · 17.4 +37.53· 215 = 200756.51 L.
Income = 6518.94 · 37+ 46.7 · 350 = 322733.82 L.
Thus, the profit from a caw will be 121977.32 L per year.

4. Conclusions

The following conclusions are attained from the sudy:

- During the process of decision-making it is becoming always more evident that it is necessary to make detailed scientific researches. Thus, the realization of livestock production necessitates the analyses of inputs in production.
- Applying Cobb-Douglas production functions gives the opportunity to realize economic analyses of farms for milk caws breeding.
- The study proved that for average production levels (17.860 kg milk per day) the most optimal structure would be: 56.8 % moisture food, 29.43 % dry food, 13.56 % concentrate and 0.21 % mineral salts.

If the theoretical arguments concerning the relative effectiveness of different economic systems are subject to empirical testing, it is necessary to do some current estimates of effectiveness indicators [13]. In the general case, it is showed that maximum income is obtained for the same input amount where the maximum profit is reached [14]. In conclusion, base on our country’s conditions, volume system nutrition is prefered

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